Problem 1. (3 points) Evaluate \( \int \frac{dx}{\sqrt{x^2 + 1}} \)

\[
\int \frac{dx}{\sqrt{x^2 + 1}} = \arctan(x) + C
\]

Let \( x = \tan(u) \), then \( dx = \sec^2(u) \, du \)

\[
\int \frac{\sec^2(u) \, du}{\sqrt{\tan^2(u) + 1}} = \int \frac{1}{\cos(u)} \, du = \arctan(x) + C
\]

Problem 2. (3 points) Evaluate \( \int \frac{x^2}{\sqrt{2x^3 + 1}} \, dx \)

\[
\int \frac{x^2}{\sqrt{2x^3 + 1}} \, dx = \frac{1}{3} \int \frac{x^3}{\sqrt{x^3 + 1}} \, dx
\]

Let \( x = \tan(u) \), then \( dx = \sec^2(u) \, du \)

\[
\int \frac{\sec^2(u) \, du}{\sqrt{\tan^3(u) + 1}} = \frac{1}{3} \int \frac{1}{\cos(u)} \, du = \frac{1}{3} \arctan(x) + C
\]

Problem 3. (4 points) Use the trigonometric substitution to evaluate \( \int \frac{dx}{\sqrt{16 + 9x^2}} \)

Let \( x = \frac{2}{3} \tan(u) \), then \( dx = \frac{2}{3} \sec^2(u) \, du \)

\[
\int \frac{\sec^2(u) \, du}{\sqrt{1 + \frac{4}{9}}} = \int \frac{2}{3} \sec(u) \, du = \frac{2}{3} \ln|\sec(u) + \tan(u)| + C
\]

Problem 4 a. (10 points) Find an integral formula for the area of the curve

\[
y = 3x^3 + 1, \quad 0 \leq x \leq 1
\]

\[
\text{Area} = \int_0^1 (3x^3 + 1) \, dx = \left[ x^4 + x \right]_0^1 = 2
\]

b. (10 points) Find an integral formula for the surface area of the curve in part (a) rotated around the x-axis. Simply the integrand, and evaluate the integral.

\[
\text{Surface Area} = 2\pi \int_0^1 \sqrt{1 + (3x^3 + 1)^2} \, dx
\]

Problem 5 a. (7 points) Sketch the graph of \( y = x^2 + 2x + 1 \). Show how many times the curve crosses the x-axis by locating \( x = 1 \) on the first time, and mark these points with \( x \). (Your sketch should not be accurate to scale.)

b. (3 points) On your picture, shade the region \( 0 \leq x \leq 1, y \geq 0, y \leq 2x^2 + 2x + 1 \) and find its area.

\[
\text{Area} = \int_0^1 (2x^2 + 2x + 1) \, dx = \left[ \frac{2}{3}x^3 + x^2 + x \right]_0^1 = \frac{5}{3}
\]